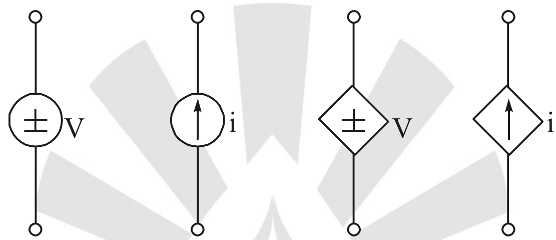
DETAILS EXPLANATIONS

EE : Paper-1 (Paper-1) [Full Syllabus]

[PART : A]

1. Independent voltage and current sources have been shown in figure (a) and (b) where the strength of voltage or current is not changed by any variation in the connected network but in case of dependent sources voltage and current for any changes in connected network.



(a) Independent Voltage Source (b) Independent Current Source (c) Dependent Voltage Source (d) Dependent Current Source

2. Synchrnous speed is the speed of rotating magnetic field produced when poly phase currents are applied to a polyphase winding. Synchronous speed N_s is given by $N_s = \frac{120f}{P}$ rpm is show that N_s pepends on the frequency of supply given to polyphase windings and the number of poles for which polyphase winding is designed.

3. Under compound DC generator has good VR. Differential compound DC generator has poor VR. Overcompound DC generator has negative VR shunt, undercompound and differential compound generator has positive VR level Compound DC generator has Zero VR.

4. Good commutation achieved through the use of interpoles is called voltage commutation. All modern machines of larger sizes attain voltage commutation by the use of compoles.

5. The function of non-volt release is to stop the DC motor. This is achieved by bringing the starter handle back to off position in case magnetic force of no-volt release or hold coil, becomes less than the spiral-spring force.

6. During commutation, current in the commutated coil reverses. The delayed commutation in DC machines is caused by emf induced in the commuated coil (i) by self-inductance of the coil itself and (ii) by the mutual flux produced by the neighbouring coils.

7. The materials used for control rods must have very high absorption capacity for neutrons. The commonly used materials for control rod are cadmium, boron or hafnium.

8. Diversity factor is defined as the ratio of the sum of the individual maximum demands to maximum demand of the group whereas load factor is defined as the ratio of average load to the maximum demand.

9. The factors affecting corona loss are system frequency, system voltage, air conductivity, air density, conductor diameter, conductor surface condition, atmospheric conditions and load current.

10. The regulation is greater than voltage drop in a medium/ long transmission line due to rise in voltage owing to Ferranti effect.

11. A distortion line is one in which the attenuation constant α is frequency independent while phase constant β is linearly dependent on frequency from the general expression for α and β.

$\gamma = \alpha + j\beta = \sqrt{(R + + j\omega L)(G + j\omega C)}$

a distortionless line results if the line parameters are such that

$$\frac{R}{L} = \frac{G}{C}$$

12. A fundamental cut-set a graph with respect to a tree a cut set formed by one and only are twig and a set of links. Thus, in a graph, for each twig of a chosen tree, there would be a fundamental cut. set for a graph having N nodes there will be (N – 1) fundamental cut set (i.e., equal to the number of twigs.)

As a convention, the orientation of cut set is so chosen that it coincides with the orientation of its twig.

13. ***Properties :***

- It consists of all the nodes of the graph.
- If the graph has N number of nodes, the tree will have (N–1) branches.
- There will be no closed path in the tree.
- There can be many possible different tree for a given graph depending on the number of nodes and branches.

14. ***Effect of integral Controller :***

- It adds an open loop pole at orgin.
- It increases the type and order of system by '1'.
- It decreases the study state error.
- It reduce the value fo ξ so that reduce the transient stability of the system.

15. When an electric field is applied, the positive atomic nuclei and the electrons are pulled in opposite direction in the material. Consequently, their centres are displaced in the direction of the field. This displacement is represented as an electric dipole moment here.

16. $\because$ V is constant so

$$Q = CV = \frac{\in A}{d} V$$

$$\therefore \quad \chi = \in_r - 1$$

$$= 2 - 1 = 1$$

If charge double then $\in_r$ also become 2 times.

17. $V_{big} = n^{2/3}$

$$V_{small} = (27)^{2/3} \times 2 \text{ volt} = 18 \text{ volt.}$$

18. When a transverse magnetic field is applied to a current carrying conductor bar, an electric field induced in perpendicular direction to both magnetic field and current. This effect is called Hall effect. The voltage induced between two faces of conducting bar is called Hall voltage.

19. ***Ferromagnetic Materials :***

- Ferromagnetic materials are spontaneously magnetized even in the absence of external magnetic field.
- All ferromagnetic materials becomes paramagnetic above certain temperature called as curie temperature.
- The value of magnetic susceptibility is positive and very large.
- Ferromagnetic materials are characterized by parallel allignment of dipole moment even in the absence of external field.
- Most important ferromagnetic materials are Fe (Iron), Cobalt (Co), Nickel (Ni), Gadolinium (Gd).

20. High permeability, high retentivity, high coercivity, large hysteresis loss and high curie temperature. Suitable for permanent magnet *Example:* ALNICO, Carbon Steel, Tungsten Steel.

[PART : B]

21. The total impedance is

$$Z = (4 + j2) + (15 - j10)$$
$$= 19 - j8 = 20.62\angle{-22.83^o}\Omega$$

The current through the circuit is

$$I = \frac{V_s}{Z} = \frac{220\angle 0^o}{20.62\angle -22.83^o}$$
$$= 10.67\angle 22.83^o A$$

(a) For the source, the complese power is

$$S_s = V_s I^*$$
$$= \left(220\angle 0^o\right)\left(10.67\angle -22.83^o\right)$$
$$= 2347.4\angle -22.83^o$$
$$= \left(2163.5 - j910.8\right)VA$$

From this we obtain the real power is 2163.5 W and the reactive power as 910.8 VAR (Leading).

(b) For the line, the voltage is

$$V_{line} = (4 + j2)I = (4.472\angle 26.57^o)(10.67\angle 22.83^o)$$
$$V_{line} = 47.72\angle 49.4^o\ V_{rms}$$

The complex power absorbed by the line is

$$S_{line} = V_{line}I^* = (47.72\angle 49.4^o)(10.67\angle -22.83^o)$$
$$= 509.2\angle 26.57^o = 455.4 + j227.7\ VA$$

or $S_{line} = |I^2|Z_{line} = (10.67)^2(4 + j2) = 455.4 + j227.7$ VA

That is the real power is 4.554 W and the reactive power is 227.76 VAR(lagging).

22. Incase of three-phase synchronous motor, electromagnetic torque is given by $T_e \propto$ (stator field strength)(Rotor field strength) sin δ

where, δ = torque angle between stator pole and rotor poles. With the rotor stationary, when 3-phase supply is given to stator, a rotating magnetic field is set up. As a result, the torque angle δ between ratating stator poles and stationary rotor poles varies with time. Let this angle δ as a function of time be expressed as δ = wt, then

Torque a (stator field strength)(rotor field strenght) sin wt.

The toreque given by above expression varies sinsoidally with time. Its value reverses during each half cycle; average value of this torque over a complete cycle is zero thus a synchronous motor has no self-starting torque.

23. In per unt system

$V_t I_a \cos\theta = \text{Power}$

or $1 \times I_a \times 0.8 = 0.9$

$\therefore \quad I_a = 1.125$ Pu

We know that wen $r_a = 0$

$$\tan(\delta + \theta) = \frac{I_a x_q + V_t \sin\theta}{V_t \cos\theta}$$

$$\tan(\delta + \theta) = \frac{1.125 \times 0.6 + 1 \times 0.6}{1 \times 0.8}$$

$\therefore \quad (\delta + \theta) = 57.894^o$

$\delta = 57.894^o - \cos^{-1}(0.8) = 57.894^o - 36.87^o = 21.024^o$

Also $\quad I_d = I_a \sin(\delta + \theta) = 1.125 \sin(57.894^o) = 0.953$ pu

$\therefore \quad E_f = V_t \cos\delta + I_d x_d$

$= 1 \times \cos 21.024^o + 0.953 \times 1.0$

$= 1.8864$ pu

When loss of excitation takes place, $e_f = 0$ and the maximum power is then given by

$$\frac{1}{2} V_t^2 \left(\frac{1}{x_q} - \frac{1}{x_d}\right) \sin 2\delta = \frac{1}{2}\left(\frac{1}{0.6} - \frac{1}{1}\right) \sin 90^o = 0.333 \text{ pu}$$

with loss of excitation, the maximum power that the reluctance generator can deliver to infinite bus is 0.333 pu. As this less than 0.9 pu the generator will lose synchronism.

24. Stator field rotates at synchronous speed

$$n_s = \frac{2f_1}{P} \text{ rps}$$

Rotor speed = Speed of rotor with respect to stator = n_r rps

Frequency of rator current = sf_1 Hz

Speed of rotor field with respect to rotor

$$= \frac{2sf_1}{p} = s.n_s \text{rps}$$

$\therefore$ Speed of stator field wrt stator = speed of rotor field wrt rotor=n_s

This shows rotating fields of the stator and rotor are stationary wrt each other.

25. The impedance drop in a line is the voltage drop in the line impedance and is equal to the phase difference between the sending end voltage and the receiving end voltage. The voltage drop in a line is the arithmetical difference between the sending-end and receiving end voltages. The voltage regulation is defined as the change in voltage at the receiving end when full load is thrown off, the sending end voltage being held constant.

26. The part of power system which distributes electric power for local use is known as distribution system.

In general, the conductor system, by means of which electrical power is conveyed from bulk power source or sources (generating stations or major substations supplied over transmission lines) to the consumers is known as distribution system. It generally consists of feeders distributors and service mains, as shown in figure.

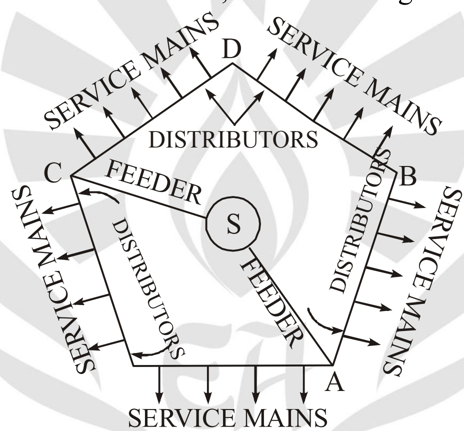


Figure : Distribution Systems

27. It is disadvantageous to provide either too high sag or too low sag owing to the following facts.

In case the sag is too high, more conductor material is required, more weight on the supports is to be supported, higher supports are necessary and there is a chance of greater swing-amplitude due to wind. On the other hand in the case of too low sag, there is more tension in the conductor and thus the conductor is liable to break if any additional stress is to be taken, such as due to vibration of line or due to fall in temperature.

28. A pin or paddle wheel as a 'curl meter' the force exerted on the each blade of the paddle wheel, the force being proportional to the component of the field normal to the surface of that blade. To test and field for curl we dip one paddle wheel into the field with the axis of the paddle wheel lined up with the direction of the component of curl desired. No rotation means no curl larger angular velocities mean greater values of the curl a reverse in the direction of spin means a reversal in the sign of the curl. In order find the direction of vector curl, we should place one paddle wheel in the field and hunt around for the orientation which produces the greatest torque. the direction of the curl is then along the axis of the paddle wheel.

29. Polarization, $\vec{P} = N\vec{p}$

$$= 2 \times 10^{19} \times 1.8 \times 10^{-27}\vec{a}_x$$

$$= 3.8 \times 10^{-8}\vec{a}_x \;\; C/m^2$$

$$\vec{P} = \epsilon_o(\epsilon_r - 1)\vec{E}$$

$$3.8 \times 10^{-8} = 8.85 \times 10^{-12} \times 10^5(\epsilon_r - 1)$$

$$\epsilon_r - 1 = 0.043$$

$$\epsilon_r = 1.043$$

30. $A = 2.1 \times 10^{-6}\ m^2$

$I = 20\ A$

$n = 8.5 \times 10^{28}$ electrons/m^3

$e = 1.6 \times 10^{-19}C$

Current, $I = ne\ AV_d$

$$\Rightarrow \quad V_d = \frac{I}{neA} = \frac{20}{8.5\times10^{28}\times1.6\times10^{-19}\times2.1\times10^{-6}}$$

$= 0.7 \times 10^{-3}$ m/sec $= 0.7$ mm/sec

31. Comparing $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$ with

$$s^2 + s2\xi\omega_n + \omega_n^2 = 0$$

$$\omega_n = \frac{1}{\sqrt{LC}} \text{ and } 2\xi\omega_n = \frac{R}{L}$$

$$\Rightarrow \quad \xi = \frac{R}{2L\omega_n} \Rightarrow \xi = \frac{R\sqrt{LC}}{2L\times1} = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$\because \quad \xi < 1$ (under damped)

$$\frac{R}{2}\sqrt{\frac{C}{L}} < 1 \Rightarrow R < 2\sqrt{\frac{L}{C}}$$

32. Let a DC voltage V be applied suddenly (i.e. at t = 0) by closing a switch k in a series RL circuit as shown in figure.

Applying (kVL) yields

$$V = Ri + L\frac{di}{dt} \qquad ...(1)$$

Taking laplace transform at both side of equation(1)

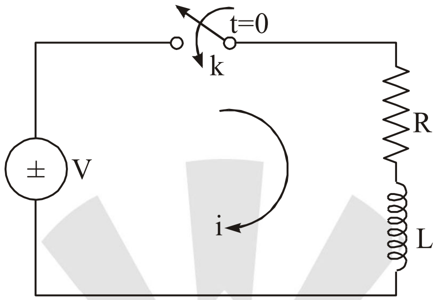


Figure : Series RL Circuit

$$\frac{V}{s} = Ri_s + L(Si_s - i(0^-))$$

Initial charging current $i(0^-) = 0$

$$\frac{V}{s} = Ri_s + Lsi_s \qquad ...(2)$$

$$\frac{V}{s} = (R + L_s)\, i_s$$

$$\Rightarrow \qquad i_s = \frac{V}{s(R+L_s)} = \frac{V}{Ls(R/L+s)}$$

$$i_s = \frac{V}{Rs} - \frac{V}{R(R/L+s)} \qquad ...(3)$$

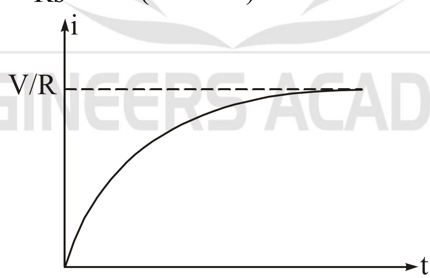


Figure : Charging Current Profile

Take inverse laplace of equation (3) both side

$$\left[i_{(t)} = \frac{V}{R} - \frac{V}{R}e^{-\frac{R}{L}t} \right]$$

where, τ = Time constant = $\frac{L}{R}$

[PART : C]

33. We determine the transform impedance and admittance representations for each of the elements and initial condtion sources.

Resistance : For resistance, the voltage and current are related in the time domain by ohm's law.

$$V_R(t) = R\, i_R(t)$$

or $$i_R(t) = GV_R(t)$$

$$G = \frac{1}{R} \qquad ...(1)$$

The corresponding transform equation are

$$V_R(s) = RI_R(s)$$

$$I_R(s) = GV_R(s) \qquad ...(2)$$

The ratio of transform voltage $V_R(s)$ to the transform current $I_R(s)$ is defined as the transform impedance of the resistor, expressed as

$$Z_R(s) = \frac{V_R(s)}{I_R(s)} = R \qquad ...(3)$$

Similarly, the reciprocal of this ratio is the transform admittance for the resistor, expressed as

$$Y_R(s) = \frac{I_R(s)}{V_R(s)} = G \qquad ...(4)$$

From the above results, we can say that the resistor is frequency insensitive to the complex frequency.

Figure (a) shows a network representing resistor R current $i_R(t)$ and Voltage $V_R(t)$ in time domain.

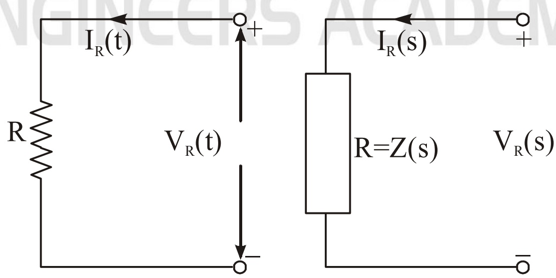


Figure (a) *Figure (b)*

Figure (b) gives the network representation of the same resistor and also transforms current $I_R(s)$ and voltage $V_R(s)$.

Inductance :

The time domain relation between the current in inductance $i_L(t)$ and the voltage $V_L(t)$ across it is expressed as

$$V_L(t) = L\frac{di_L(t)}{dt}$$

and $$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} V_L(t)\,dt \qquad ...(5)$$

The equivalent transform equation for the voltage expression is

$$V_L(s) = L[sI_L(s) - i_L(0^+)] \qquad ...(6)$$

$$\Rightarrow \quad LsI_L(s) = V_L(s) + Li_L(0^+) \qquad ...(7)$$

In the equation (5) and (6) $V_L(s)$ is the transform of the applied voltage $V_L(t)$ and $i_L(0^+)$ is the transform voltage caused by the initial current $i_L(0^+)$ present in the inductor at time $t = 0^+$.

Considering the sum of the transform voltage and the initial current voltage as $V_L(s)$ we have the transform impedance for the inductor.

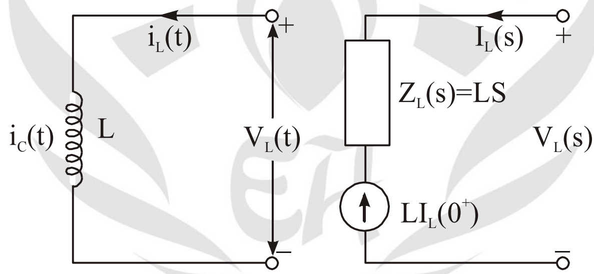


Figure (a) *Figure (b)*

$$Z_L = \frac{V_l(s)}{I_L(s)} = SL \qquad ...(8)$$

Figure (b) gives the transform representation of same inductor in terms of impedance using equation (6). The transform equation for the current expression of equation (7) is

$$I_L(s) = \left[\frac{V_L(s)}{s} + \frac{\int_{-\infty}^{0^+} V_L(t)\,dt}{s}\right]\frac{1}{L} \qquad ...(9)$$

But $\int_{-\infty}^{t} V_L(t)\,dt = Li_L(0^+)$ ...(10)

Hence equation (9) be comes

$$I_L(s) = \frac{1}{L}.\frac{V_L(s)}{s} + \frac{i_L(0^+)}{s} \quad ...(11)$$

or $$\frac{1}{Ls}.V_L(s) = I_L(s) - \frac{i_L(0^+)}{s} \quad ...(12)$$

in the above equation $\frac{i_L(0^+)}{s}$ is the transform caused by the initial current $i_L(0^+)$ in the inductor.

Let, $$I_1(s) = I_L(s) - \frac{i_L(0^+)}{s} \quad ...(13)$$

then the equation (12) becomes

$$\frac{1}{Ls}V_L(s) = I_1(s) \quad ...(14)$$

Where $I_1(s)$ is the total transform current through the indictor L. The transform impedance becomes

$$Y_L(s) = \frac{I_1(s)}{V_L(s)} = \frac{1}{Ls} \quad ...(15)$$

Figure (a) shows the time domain representation of inductor L with inital current $I_L(0^+)$ figure (b) shows equivalent transfor circuit theus contains an admittance of value $\frac{1}{Ls}$ and equivalent transform current source.

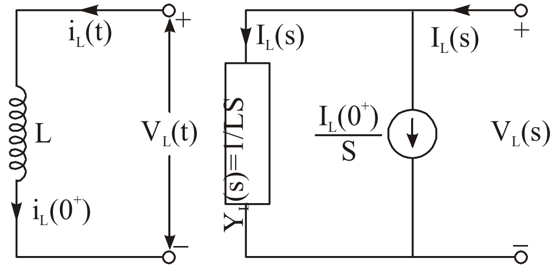


Figure (a) *Figure (b)*

Capacitance :

The time domain relation between voltage and current expressed as

$$i_c(t) =- C\frac{dV_c(t)}{dt} \qquad ...(16)$$

The equivalent transform equation for the voltage expression is

$$V_C(s) = \frac{1}{C}\left[\frac{I_c(s)}{s}+\frac{Q(0^+)}{s}\right] \qquad ...(17)$$

where $\frac{Q(0^+)}{s} = V_c(0^+)$ is the initial voltage across the capacitor. The above equation becomes

$$\frac{1}{Cs}I_C(s) = V_C(s) - \frac{V_C(0^+)}{s} \qquad ...(18)$$

By considering the initial charge on the capacitor zero. The equation becomes

$$\frac{1}{C(s)}I(s) = V(s)$$

The transform impedance of the capacitor is the ratio of transform voltage V(s) to the transform current I(s) and is

$$Z(s) = \frac{V(s)}{I(s)} = \frac{1}{CS}$$

the transform admittance of the capacitor is the ratio of transform current I(s) of transform voltage V(s) is

$$Y(s) = \frac{I(s)}{V(s)} = CS$$

34. The transformer core loss P_i has two components namely hystersis loss p_n and eddy current loss p_e

$$p_i = p_n + p_e$$

$$p_i = k_n f B_m^n + k_e f^2 B_m^2 \qquad (1)$$

Where k_n = A constant whose value defends upon the ferromagnetic material

B_m = Maximum value of the flux density

f = Supply frequency

The exponent n varies in the range 1.5 to 2.5 depending upon the material, for a given B_m the hysteries loss varies directly as the frequency and edddy current loss varies as the square of the frquency, that is

$$p_n \propto f \text{ or } p_n = af$$

and $p_e \propto f^2$ or $P_e = bf^2$

Where a dnd b are constant

$$\therefore \quad p_i = af + bf^2 \qquad ...(2)$$

For separation of these two losses the no-load test is performed on the transformer. However, the primary of the transformer is connected to a variable frequency and variable sin soidal supply and the secondary is open circuited

Now $V \simeq E = 4.44\ f\ \phi_m T$

or $\dfrac{V}{f} = 4.44\ B_m A_i T$

for any transformer T and A_i are constant therefore B_m will remain constant if the test is conducted so that ratio $\left(\dfrac{V}{f}\right)$ is keft constant.

From equation (2) we get

$$\frac{p_i}{f} = bf + a \qquad ...(3)$$

During this test, the applied voltage V and frequency 'f' are varied together so that $\left(\dfrac{V}{f}\right)$ is keft constant. The core loss is obtained at different frequencies. A graph of $\left(\dfrac{p_i}{f}\right)$ versus frequency 'f' is plotted. This graph is a straight line AB of the form y = mx + c, as show in figure.

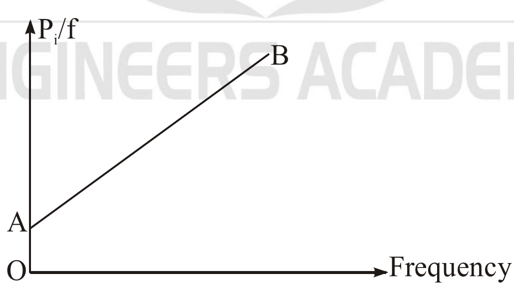


Figure : Variation of (p_i/f) with f

The intercept of the straight line on the vertical axis given a and the slope of the line AB gives b. Thus knowing the constants a and b, hystersis and eddy-current losses can be separated.

35. $r = \frac{20}{2} = 10 \text{ mm} = 0.01 \text{ m}$

Spacing between conductors

$$d_1 = AB = 3 \text{ m}$$
$$d_2 = BC = 3 \text{ m}$$
$$d_3 = CA = 6 \text{ m}$$

Capacitance per phase per m length,

$$C_N = \frac{2\pi \in_0}{\log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r}}$$

$$C_N = \frac{2\pi \times 8.854 \times 10^{-12}}{\log_e \frac{\sqrt[3]{3\times3\times6}}{0.01}} = 9.3737 \times 10^{-12} \text{ F } \textbf{\textit{Ans.}}$$

Charging current per phase,

$$I_C = 2\pi f C_N V_{ph}$$

$$= 2\pi \times 50 \times 9.3737 \times 10^{-12} \times \frac{220\times1000}{\sqrt{3}} = 0.374 \text{ mA } \textbf{\textit{Ans.}}$$

36. (i) Equivalent T-network in which Shunt admittance

$$Y = C = 0.001376\angle 90.4^o \text{ S } \textbf{\textit{Ans.}}$$

and impedance,

$$Z = \frac{2(A-1)}{C} = \frac{2\left(0.945\angle 1.02^o - 1\right)}{0.001376\angle 90.4^o}$$

$$= 2\times\frac{-0.0552 + j0.01682}{0.000688\angle 90.4^o} = 167.75\angle 72.6^o\ \Omega \textbf{\textit{ Ans.}}$$

(ii) Equivalent π-network in which Series impedance,

$$Z = B = 82.3\angle 73.03^o\ \Omega \textbf{\textit{ Ans.}}$$

and shunt admittance,

$$Y = \frac{2(A-1)}{B}$$

$$= \frac{2\left(0.945\angle 1.02^o - 1\right)}{82.3\angle 73.03^o}$$

$$= \frac{2\times0.05766\angle 163^o}{82.3\angle 73.03^o} = 0.001401\angle 89.97^o \text{ S } \textbf{\textit{Ans.}}$$

37. For a long transmission line terminals at load impedance (Z_L)

Here $dI = y\,dx\,V$

and $dV = ZI\,dx$

$$\frac{dV}{dx} = ZI \text{ and } \frac{dI}{dx} = yV$$

$$\frac{d^2V}{dx^2} = Z\frac{dI}{dx} = ZyV$$

$$\frac{d^2V}{dx^2} - yZV = 0$$

Let $yZ = r^2$ then $\frac{d^2V}{dx^2} - r^2V = 0$

Solution of differential equation

$$V = V_+e^{-rx} + V_-e^{-rx}$$

$$I = I_+e^{-rx} + I_-e^{rx}$$

At (x = 0) $Z_L = \frac{V}{I} = \frac{V_+ + V_-}{I_+ + I_-} = \frac{Z_0(I_+ - I_-)}{I_+ + I_-}$

$$\frac{I_-}{I_+} = \frac{Z_0 - Z_L}{Z_0 + Z_L} = \Gamma_R = \frac{V_-}{V_+}$$

Where, $|\Gamma_R|$ = Reflection coefficient

Z_0 = Characteristic impedance

Standing ware Retro :

Ratio of maximum value of minimum amplitudes of the voltage or current waves.

$$SWR = \frac{1+|\Gamma_R|}{1-|\Gamma_R|}$$

$$Z_L = 500 + j300\ \Omega$$

$$Z_0 = 600\ \Omega$$

then, $\Gamma_R = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-100+300j}{1100+300j} = 0.277\angle 93^o$ $(\therefore\ |\Gamma_R| = 0.277)$

$$SWR = \frac{1+|\Gamma_R|}{1-|\Gamma_R|} = \frac{1.27}{0.72} = 1.77$$

38. The state of material at which resistivity reduce to zero is called super conductivity. The temperature at which there is transition from normal state to super conducting state is called transition temperature. Above critical temperature (T_c) the material is in the familiar normal state but below (T_c) it enters an entirely different super conducting state, resistance of these materials in the super conducting state is at least 10^{16} times smaller than their room temperature values.

The zero resistivity and perfect diamagnetism.

- ***Properties :***
  - (i) At room temperature, the resistivity 'ρ' of super conducting materials are greater than other elements.
  - (ii) All thermo electric effects disappear in super conducting state.
  - (iii) When a sufficient strong magnetic field is applied to super-conductor below critical temperature T_c, its super conducting property is destroyed.
  - (iv) When current is passed through the super conducting material the heat loss (I^2R) is zero.
  - (v) The magnetic flux density in super conductor is zero.

39. ***The Thevenin's theorem Statement :***

A linear active bilateral network can be replaced at any two of its terminals by an equivalent voltage source (thevenin's voltage source), V_{oc}, in series with an equivalent impedance (the venin's impedance), Z_{th}. Here V_{oc} is the open circuit voltage between the two terminals under the action of all source and initial conditions and Z_{th} is the impedance obtained across the terminals with all sources removed by their internal impedance and intial conditions reduced to zero.

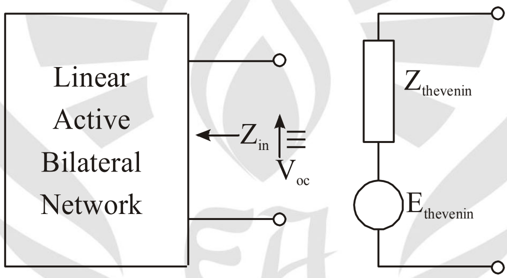


Figure : Illustration of Thevenin's theorem

Proof :

We consider a linear active circuit of figure (a) on external current source is applied through the terminals a-b where we have access to the circuit.

We have to prove that the V-i relation at termals a-b of figure (a) is identical with that of the thevenin's equivalent circuit of figure(b).

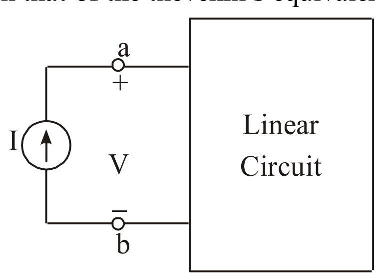


Figure : (a) Current driven Circuit

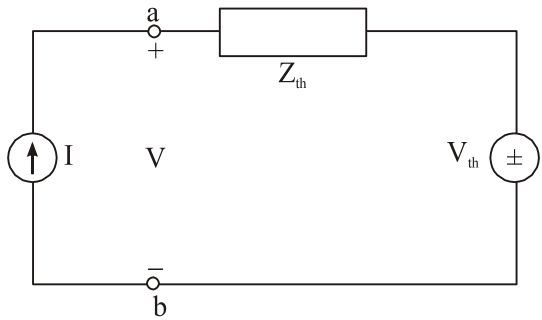


Figure : (b) The Venin's Equivalent Circuit

For simplicity, we assume that the circuit contains two independent voltage source V_{S1} and V_{S2} and two independent current sources I_{S1} and I_{S2}.

Considering the contribution due to each independent source including the external one, the voltage at a-b, V is by superposition theorem.

$V = k_oI + k_1V_{S1} + k_2V_{S2} + k_3I_{S1} + kI_{S2}$

where, k_o, k_1, k_2, k_3, k_4 are constant or

$$V = k_oI + P_o \quad ...(1)$$

Where $P_o = k_1V_{S1} + k_2V_{S2} + k_3I_{S1} + k_4I_{S2}$ = total contribution due to internal independent sources to evaluate the constant k_o and p_o of equation (1), two conditions are

- When the terminals a and b are open-circuited

$$I = 0 \text{ and } V = V_{oc} = V_{th}$$

from equation (1)

$$V_{th} = V_{oc} = P_o \Rightarrow (V_{th} = P_o)$$

- When all the internals sources are turnoff.

$P_o = 0$ and the equivalent impedance is Z_{th}.

from equation (1)

$$V = k_oI$$

or $$\frac{V}{I} = k_o = Z_{th} \Rightarrow k_o = Z_{th}$$

Thus, substituting the values of k_o and P_o the V - i relation becomes,

$$V = Z_{th}I + V_{th}$$

This represents the V-i relationship of figure (b) So, thevenin's theorem proved.

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